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SHORTER COMMUNICATION

HEAT TRANSFER PREDICTIONS FROM MASS TRANSFER MEASUREMENTS AROUND A SINGLE CYLINDER IN CROSS FLOW

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NOMENCLATURE

Δc, concentration difference [lb mole/ft^3];
G, mass velocity [lb/ft^{2}h];
h, heat transfer coefficient [Btu/h ft^2 deg F];
h_{mc}, mass transfer coefficient [lb mole/h ft^2 unit Δc];
h_{mp}, mass transfer coefficient [lb mole/h ft^2 unit p];
h_{mp}, mass transfer coefficient [lb mole/h ft^2 unit p];
MW, molecular weight of air, taken as 128·2;
N, erosion rate [lb mole/h ft^2];
p, partial pressure of naphthalene [lbf/ft^2];
P, atmospheric pressure [lbf/ft^2];
R, universal gas constant [ft lbf/lb mole °R];
T, absolute temperature [°R];
u_∞, free stream velocity, [ft/h];
x, distance from the front stagnation point;
θ, angle (degrees) from the front stagnation point;
ϕ, St/St_m;
β, wedge flow included angle.

Dimensionless numbers
M, $(\beta/\pi)/[2 - (\beta/\pi)]$;
Nu, Nusselt number;
Pr, Prandtl number;
Re, Reynolds number;
Sc, Schmidt number;
St, Stanton number;
St_m, mass Stanton number.

1. INTRODUCTION

SOME recommended empirical relationships for perimeter mean heat transfer coefficients of single heated cylinders in air (taking $Pr = 0·7$) are given in Table 1. Individual values are up to 100 per cent greater than the virtually identical expressions [1] and [2] suggest.

Richardson uses separate terms, one associated with the laminar boundary layer extending 80° each side of the front stagnation point incorporating $Re^{\frac{1}{2}}$ and the other incorporating $Re^{\frac{2}{3}}$ for the separated region around the remainder of the periphery. Figure 1 shows the two equations representing the limit of reasonable experimental values. The variation

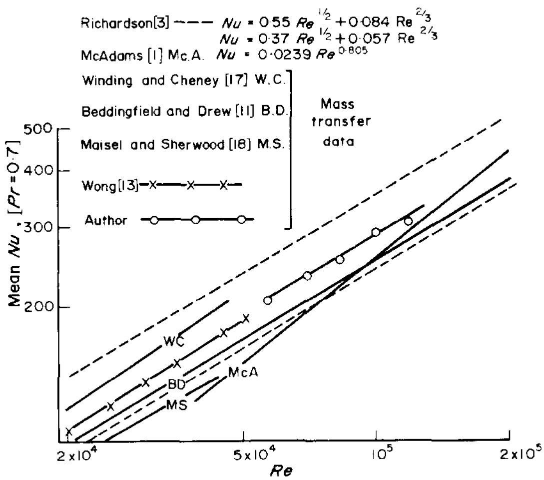


FIG. 1. Heat and mass transfer data from previous investigations.

of experimental results for apparently the same conditions is due to the wind tunnel characteristics, the history of the flow upstream and its degree of turbulence. The laminar region is affected most by free stream turbulence and the separated region, by wind tunnel blockage. A similar parallel resistance concept was suggested by White and Churchill [4].

Experimental expressions associated with the region around the front stagnation point are generally in the form recommended by Martinelli *et al.* [5] (Figs. 3 and 4)

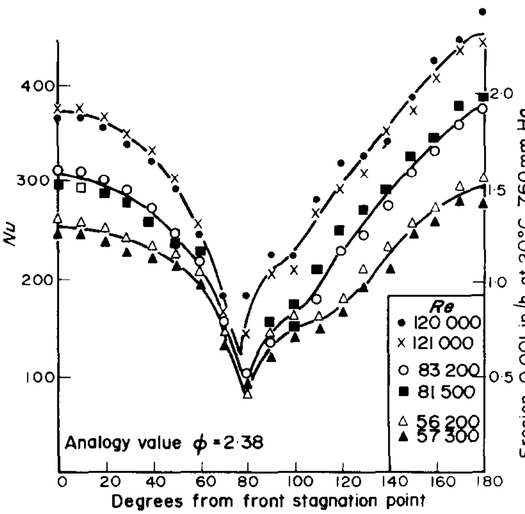
$$Nu = 1·14\ Re^{0·5}\ Pr^{0·4}\ [1 - (\theta/90)^3].$$

* The experimental work was performed while at the University of Strathclyde, Glasgow.

Theoretical attempts to predict local heat transfer coefficients have been more successful in the laminar flow region than in the separated region. Eckert [6] used wedge flow theory to obtain values 80° each side of the front stagnation point, Fig. 3. Leontev and Riagen [7] proposed the build up of a laminar boundary layer by the reversed flow each side of the rear stagnation point to obtain a theoretical curve. Consideration of the boundary layer as

Table 1. Heat transfer from a single cylinder in a cross flow of air

Author	Empirical formula	*Re* range
McAdams [1]	$Nu = 0{\cdot}0239\, Re^{0{\cdot}805}$	4×10^4–$2{\cdot}5 \times 10^5$
Kutateladze, S.S. [2]	$Nu = 0{\cdot}023\, Re^{0{\cdot}8}$	$>5 \times 10^4$
Richardson [3]	$Nu = 0{\cdot}37\, Re^{\frac{1}{2}} + 0{\cdot}057\, Re^{\frac{2}{3}}$ $Nu = 0{\cdot}55\, Re^{\frac{1}{2}} + 0{\cdot}084\, Re^{\frac{2}{3}}$	10^2–10^5



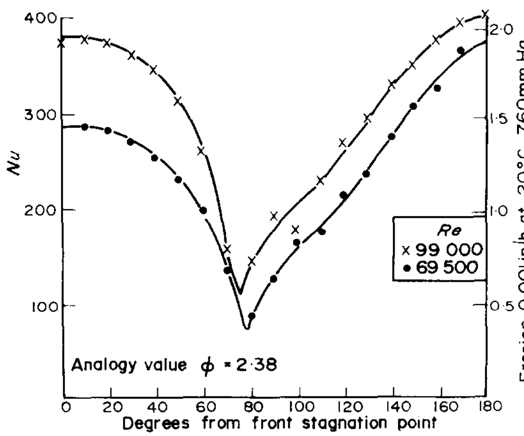


FIG. 2. *Nu* and erosion rate around cylinder periphery.

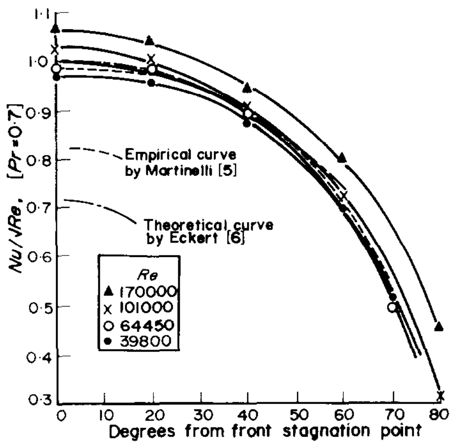


FIG. 3. Measured values by Schmidt and Wenner around the front stagnation point.

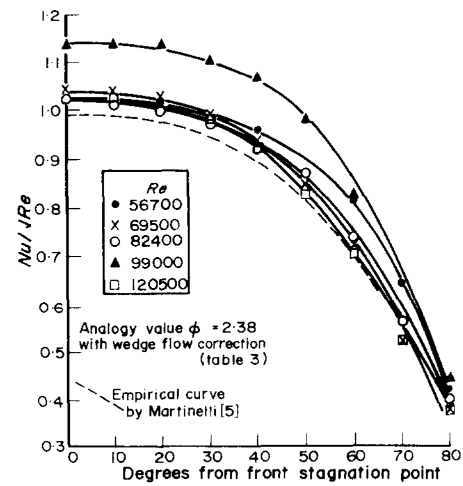


FIG. 4. Values from mass transfer experiments around the front stagnation point.

laminar in this region must be an over simplification, yet the predictions are comparable with the scattered experimental results available, Fig. 5.

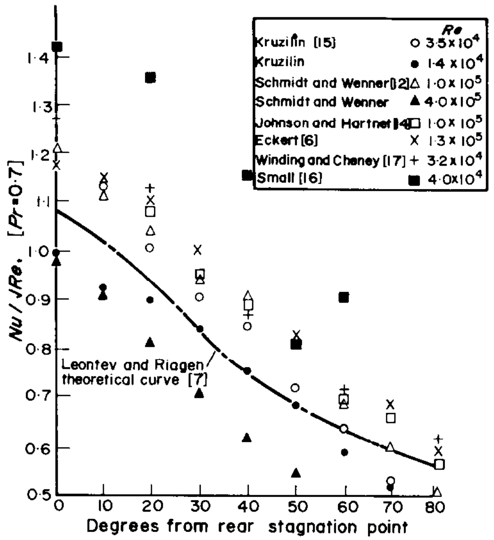


FIG. 5. Values of some previous investigations around the rear stagnation point.

Some of the many experimental results from mass transfer measurements are shown in Fig. 1. All have used the simple Chilton–Colburn Analogy [8] with $\phi = (Sc/Pr)^{\frac{2}{3}}$, to predict heat transfer coefficients from mass transfer measurements. This analogy was developed from heat-mass-momentum transfer for fully developed flow within a round pipe. Because it seems incredible that it should provide a valid heat-mass transfer analogy for flow around a cylinder, erosion measurements were made on a napthalene coated cylinder and the following analysis considered, using $Pr = 0{\cdot}7$ for air and $Sc = 2{\cdot}6$, for napthalene vapour diffusing into air.

2. A HEAT-MASS TRANSFER ANALOGY

Laminar boundary layer theory associated with a semi-infinite flat plate, with constant free stream velocity and the familiar Pohlhausen solutions have been used to solve this more complicated problem, 80° each side of the front stagnation point.

For a variable velocity u_∞ = constant x^M along a constant temperature flat plate, similarity solutions may be obtained for a series of wedge flows. Values of $Nu_x\,(Re)^{-\frac{1}{2}}$ were calculated by Eckert [6] for Pr up to 10, over wedges ranging from a plate parallel to the flow $M = 0$, to one at right angles to the flow $M = 1$. Taking $Pr = 0{\cdot}7$ and $2{\cdot}6$, values of ϕ may be calculated (Table 2).

More sophisticated approaches are reviewed by Kays [9] to give an expression.

$$\phi = 2{\cdot}37\,\sqrt{\{(1 + 1{\cdot}88\,M)/(1 + 2{\cdot}10\,M)\}}.$$

Richardson [3] suggests $Nu \propto Pr^{0{\cdot}4}$ in this laminar region, which gives $\phi = (Sc/Pr)^{0{\cdot}6}$.

Over the rear portion of the cylinder, which is subjected to a reversed flow, the boundary layer is unsteady building up and collapsing in sympathy with the vortex formation. Correlation of the wide range of experimental results and proposing a theoretical analysis are both difficult. If the flow is treated as laminar as suggested by Leontev and Riagen [7], the values of ϕ correspond to those around the front stagnation point.

Table 2. Values of ϕ for wedge flows

M	0	0·111	0·333	1·0
ϕ	2·36	2·34	2·32	2·28

Richardson [3] considers the flow as separated with $Nu \propto Re^{\frac{1}{2}}$ which agrees with the value of ϕ from the Chilton–Colburn [8] relationship. Analogy values around the cylinder circumference are summarised in Table 3.

Of the three approaches Richardson's recommendation applied to the laminar boundary layer gives rise to the largest deviation from the Chilton–Colburn value. Paradoxically a constant value of $\phi = 2{\cdot}38$, gives very reasonable results.

3. THE CALCULATION OF HEAT-TRANSFER COEFFICIENTS FROM MASS TRANSFER DATA

St_m may be easily calculated from the erosion rate, the temperature of the subliming surface and the ambient pressure

$$h_{mc} = N/\Delta c = h_{mp}RT, \quad St_m = h_{mp}RT/u.$$

The universal gas equation and the continuity equation may be used to express St_m and hence h in terms of the napthalene vapour pressure assuming that the volume diffusing from the surface is negligible compared with the free stream flow rate

$$St_m = h_{mp}P/[G(MW)], \quad h = [NP(MW)/p]\phi.$$

The vapour pressure of the napthalene is temperature dependent and of several empirical formulae which agree closely, Sherwood's [10] expression p = antilog (11·55–3765/°K) mm Hg was used.

No temperature depression of the napthalene surface could be detected throughout the tests and p values were calculated from measured values of the surrounding air temperature.

Table 3. Values of ϕ around the cylinder circumference

Analogy / θ	0	10	20	30	40	50	60	70	80	90
Chilton–Colburn [8], $\phi = 2{\cdot}38$										2·38
Richardson [3], empirical					2·2					
Laminar, theoretical, wedge flow correction	2·284	2·291	2·298	2·306	2·313	2·326	2·334	2·340	2·356	2·380

Analogy / θ	100	110	120	130	140	150	160	170	180
Chilton–Colburn [8], $\phi = 2{\cdot}38$									
Richardson [3], empirical					2·38				
Laminar, theoretical, wedge flow correction	2·356	2·340	2·334	2·326	2·313	2·306	2·298	2·291	2·284

The only test data required are the temperature and pressure of the air adjacent to the subliming surface and its erosion rate. The specific gravity of napthalene was taken as 1·14, an average of several measurements.

4. EXPERIMENTAL METHOD

Liquid napthalene was painted into a 12-in. long groove in an 18-in. long, 3-in. dia. wooden cylinder and turned on a centre lathe to give a smooth surface. The napthalene was eroded in air streams up to 80 ft/s in a wind tunnel of 5-ft dia. working section. Erosion times varied from 2–5 h, with the air velocity, temperature and pressure recorded at not more than $\frac{1}{4}$-h intervals.

Erosions were measured by means of a Ferranti micro-comparator from two sets of readings, with the cylinder between centres, before and after erosion. Readings were taken at 10° intervals at 10 sections 1-in. apart along the length of napthalene. Symmetrical flow around the cylinder enabled each erosion rate, 0–180° from the front stagnation point to be the average of twenty measured differences in surface profile.

5. EXPERIMENTAL RESULTS

Erosion measurements shown in Fig. 2 confirm the familiar heat transfer pattern. In some tests, the discontinuity between the transition region (80° < θ < 100°), where separation occurs, and the region associated with the rear stagnation point begins, ($\theta \approx 100°$), is clearly shown. Generally, there is little discrepancy between erosion measurements at close Re tests. Predicted Nu values around the front stagnation point are higher than would be expected from Martinelli's [5] empirical expression.

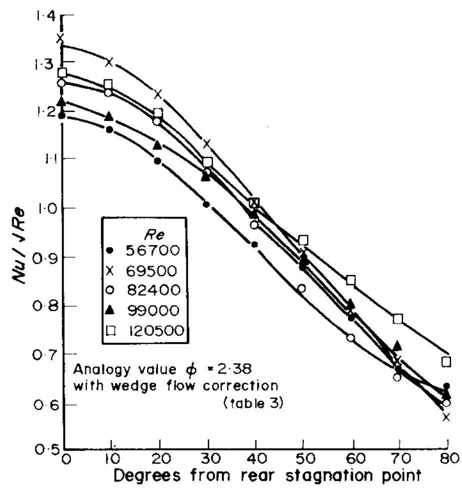


FIG. 6. Values from mass transfer experiments around the rear stagnation point.

Figure 4 shows the values of $Nu \,.\, Re^{-\frac{1}{2}}$, in the front stagnation point region, obtained from erosion measurements using an analogy value $\phi = 2{\cdot}38$ with wedge flow correction. They are similar to Schmidt and Wenner's [12] measurements shown in Fig. 3, but generally slightly greater than values indicated by the empirical curve of Martinelli [5] and Eckert's [6] theoretical curve up to $\theta \approx 60°$.

Similarly for the rear stagnation point region, values of

$Nu \,.\, Re^{-\frac{1}{2}}$ from erosion measurements are well within the range of those obtained by other investigators, by direct measurement, as shown in Figs. 5 and 6. However, Leontev and Riagen's [7] theoretical expression gives values somewhat lower than the majority of experimental results and it seems unlikely that the boundary layer around the downstream region of the cylinder is steady and laminar.

Richardson's [3] limiting empirical formulae almost envelop the range of experimental overall values of Nu as shown in Fig. 1. The idea of upper and lower limits depending on the experimental conditions is sensible, and again the test results, obtained from erosion measurements, compare favourably with those obtained by direct measurement.

6. CONCLUSIONS

(i) The mass transfer pattern around a single cylinder in cross flow is found by direct measurement of erosion to be similar to that for heat transfer, with two flow regions associated with the front and rear stagnation points.

(ii) Whether the boundary layer is treated as laminar around each stagnation point and corrected according to wedge flow theory, or, laminar only at the front, and separated around the remainder of the cylinder, according to Richardson's [3] recommendations, the corresponding analogy values differ by less than 7·5 per cent from the constant Chilton–Colburn [8] value of $\phi = 2{\cdot}38$.

(iii) The Chilton–Colburn [8] analogy may be used to predict heat transfer values from mass transfer measurements giving local and overall coefficients well within the spectrum of heat transfer data by direct measurement. The error involved is much less than the deviation caused by such factors as free stream turbulence and wind tunnel blockage.

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